Why electron collider should be linear?

- At the 2017/6/2 book club, a member of the audience asked why an electron collider should be linear while a proton collider can be circular.
- As another member in the audience, I tried to answer by arguing that it will take a stronger magnetic field to bend the electron than the proton.
- My answer is wrong because,
  - When the particle goes slow (much below light speed), it is actually easier to bend the electron than the proton if they are going at the same velocity. This is because the lighter electron has lower momentum.
  - If they are moving with the same kinetic energy, the electron still has lower momentum and will still bend more easily.
  - When they move near light speed at the same energy, their momenta become comparable. Bending them is equally difficult.
- A better way to explain why electron collider should not be circular is like this:
  - Consider electrons and protons going around in a circle and accelerated to the same energy, at speeds close to that of light, i.e., $v_e \sim v_p \sim c$.
  - The electron energy E_e is given by,

    $$E_e = \gamma_e m_e c^2,$$

    where m_e is the electron rest mass and γ_e is given by,

    $$\gamma_e = \frac{1}{\sqrt{1-\left(\frac{v_e}{c}\right)^2}}.$$
  - Similarly, the proton energy E_p is given by,

    $$E_p = \gamma_p m_p c^2$$
  - For them to have the same energy, $E_e = E_p \Rightarrow \frac{\gamma_e}{\gamma_p} = \frac{m_p}{m_e} \sim 2000$.
  - As they go round a circle of radius r at speed v, they experience a centripetal acceleration $a = \frac{v^2}{r}$.
  - A charged particle will radiate if it accelerates.
  - The radiated power P is proportional to the product of two terms, the square of the acceleration and the fourth power of γ.

    $$P \propto \left(\frac{v^2}{r}\right)^2 \gamma^4.$$
  - Since $\frac{\gamma_e}{\gamma_p} \sim 2000$, the circulating electron radiates 10^{13} times more power than the proton.
  - As a result, much of the energy delivered to the electron by the accelerator is radiated rather than going to collisional energy, making it very difficult to pump electron to high energy.
- Why, for electrons, a linear accelerator works better than a circular one? This is best answered with a numerical example,
  - Suppose we are to accelerate electrons to 1 TeV. This gives $\gamma_e \sim 2\times10^6$ or $v_e \sim 0.9999999999999c$.
  - We will compare the power radiated for the two accelerator geometries, circular versus linear.
  - For the circular accelerator of radius r, the power radiated will be proportional to,

$$P \propto 1.6\times10^{25}\left(\frac{c^2}{r}\right)^2.$$

- For the linear accelerator, we pump the electron to 1 TeV in two stages.
    - In the first stage, we get it up to 510 MeV in a circular accelerator. This gives $\gamma_e \sim 1\times10^3$ or $v_e \sim 0.9999995c$.
    - In the second and final stage, we increase it from 510 MeV to 1 TeV in a linear accelerator.
    - If the length of the linear accelerator is comparable to *r*, the power radiated in stage 2 can be shown to be,

$$P \propto 1.6\times10^{25}\left(\frac{\Delta v}{c}\right)^2\left(\frac{c^2}{r}\right)^2.$$

    - The power radiated in stage 1 can be shown to be about 6× less than stage 2.
    - So, the total power radiated is $\sim\left(\frac{\Delta v}{c}\right)^2$ or $\sim10^{-13}$ times that of the circular case.
- For 1 TeV proton going round a circle of radius *r*, the power radiated will be $(2000)^{-4}$ or $\sim10^{-13}$ times that of the electron going round the same track. So, the power radiated by the 1 TeV electron accelerated along a linear track is only about that of a 1 TeV proton in a circular track of comparable dimension.

A physicist in the audience
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